MATH 54 – HINTS TO HOMEWORK 12

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Here are a couple of hints to Homework 12! Enjoy :)

Note: This homework is *very* hard and *very* long! Make sure to start on it early! Also, make sure to look at my 'Partial Differential Equations'-Handout, it contains lots of sample problems with solutions!

IMPORTANT: For sections 10.5 and 10.6, you're **NOT** allowed to use the formulas that the book gives you, such as (14) on page 668 or (5) on page 678! You really have to do those problems **from scratch** and you will be heavily penalized if you don't do this! Also, you're not allowed to use your calculator on this, you really have to evaluate any integrals you find! Ditto with just 'copying' the answer from the back of the book!

SECTION 10.2: METHOD OF SEPARATION OF VARIABLES

10.2.1, 10.2.3, 10.2.4. Just solve your equation the way you would usually do, as in chapter 4 (i.e. find the auxiliary equation and plug into your intiial conditions). You may or may not find a contradiction! If you find $0 = 0$, that usually means there are infinitely many solutions, depending on your constant A or B. For $10.2.4$, remember to differentiate your answer once before you plug in $t = \pi$ for $y'(\pi) = 0$

10.2.9. You have to split up your analysis into three cases:

Case 1: $\lambda > 0$. Then let $\lambda = \omega^2$, where $\omega > 0$. This helps you get rid of square roots.

Case 2: $\lambda = 0$.

Case 3: $\lambda < 0$. Then $\lambda = -\omega^2$, where $\omega < 0$.

In each case, solve the equation and plug in your initial condition. You may or may not get a contradiction. Also, remember that y has to be nonzero!

10.2.27. This is just separation of variables. Plug $u(r, \theta) = R(r)T(\theta)$ into your equation, put all the terms with T on the left-hand-side, and all the terms with R on the right-hand-side, and use the fact that the $LHS = RHS = \lambda$, where λ doesn't depend on r or θ.

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SECTION 10.3: FOURIER SERIES

10.3.1 − 10.3.6. What a fun problem :) f is even if $f(-x) = f(x)$, f is odd if $f(-x) = -f(x)$.

10.3.7. Just calculate $(fg)(-x) = f(-x)g(-x)$

10.3.9, 10.3.12, 10.3.13. For what follows, use the following formulas (or see the examples in lecture):

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{T}\right) + b_n \sin\left(\frac{n\pi x}{T}\right) \right\}$$

$$a_n = \frac{1}{T} \int_{-T}^{T} f(x) \cos\left(\frac{n\pi x}{T}\right) dx$$

$$b_n = \frac{1}{T} \int_{-T}^{T} f(x) \sin\left(\frac{n\pi x}{T}\right) dx$$

Where $T = \pi$ in 10.3.9, $T = 2$ in 10.3.11, $T = 1$ in 10.3.13. See also the remarks below! Also, to evaluate the integrals, it is best to use **tabular integration** (see handout) or integration by parts if the integral is easy to evaluate.

Note: Don't worry *too* much about 10.3.11, it won't appear on the exam!

10.3.9. Notice f is odd, so all the a_n are 0.

10.3.11. To calculate the integral, split up the integral from $\int_{-\pi}^{0} + \int_{0}^{\pi}$

10.3.17, 10.3.19. The Fourier series converges to $f(x)$ if f is **continuous** at x, and converges to $\frac{f(x^+)+f(x^-)}{2}$ if f is **discontinuous** at x. As for the endpoints T and $-T$, the fourier series converges to the average of f at those endpoints.

10.3.28. This problem is awesome!!! :D I'll hope you'll find it as exciting as I do :)

(a) Calculate a_n using the formula above with $T = \pi$

(b) Plug in $x = 0$ in the formula in (a). Notice that since f is continuous at 0, the Fourier series of f, i.e. the sum on the right-hand-side converges to $f(0) = 0$, so:

$$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n0) = 0$$

Then the rest is just some algebra!

(c) Ditto, but this time you plug in $x = \pi$. Here you have to use the fact that the sum on the right-hand-side converges to $\frac{\pi+\pi}{2} = \pi$. Also use the fact that $\cos(n\pi) = (-1)^n$ and $(-1)^n(-1)^n = 1$

10.3.29. This is just 'hugging', i.e.:

$$a_0 = \frac{f \cdot P_0}{P_0 \cdot P_0}, \qquad a_1 = \frac{f \cdot P_1}{P_1 \cdot P_1} \qquad a_0 = \frac{f \cdot P_2}{P_2 \cdot P_2}$$

where $f \cdot g = \int_{-1}^{1} f(t)g(t)dt$

SECTION 10.4: FOURIER COSINE AND SINE SERIES

IMPORTANT NOTE: The book uses the following trick **A LOT**:

Namely, suppose that when you calculate your coefficients A_m or B_m, you get something like: $A_m = \frac{(-1)^{m+1}+1}{\pi m}$.

Then notice the following: If m is even, then $(-1)^{m+1} + 1 = 0$, so $A_m = 0$, and if m is odd, $(-1)^{m+1} + 1 = -2$, and $A_m = \frac{-2}{\pi m}$.

So at some point, you would like to say:

$$f(x) \text{``}=\text{''} \sum_{m=1, m odd}^{\infty} A_m \cos(mx)$$

The way you do this is as follows: Since m is odd $m = 2k - 1$, for $k = 1, 2, 3 \cdots$, and so the sum becomes:

$$f(x) \text{``}=\text{''} \sum_{k=1}^{\infty} \frac{-2}{\pi(2k-1)} \cos((2k-1)x)$$

10.4.1, 10.4.3. π-periodic extension just means 'repeat the graph of f'.

The even-2π periodic extension is just the function:

$$f_e(x) = \begin{cases} f(-x) & \text{if} -\pi < x < 0 \\ f(x) & \text{if} 0 < x < \pi \end{cases}$$

The odd-2π periodic extension is just the function:

$$f_o(x) = \begin{cases} -f(-x) & \text{if} -\pi < x < 0 \\ 0 & \text{if} x = 0 \\ f(x) & \text{if} 0 < x < \pi \end{cases}$$

And repeat all those graphs!

10.4.5, 10.4.7. Use the formulas:

$$f(x) \text{``}=\text{''} \sum_{m=1}^{\infty} B_m \sin\left(\frac{\pi m x}{T}\right)$$

where:

$$B_0 = 0$$

$$B_m = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{\pi m x}{T}\right)dx$$

10.4.11, 10.4.13. Use the formulas:

$$f(x) \text{``}=\text{''} \sum_{m=0}^{\infty} A_m \cos\left(\frac{\pi m x}{T}\right)$$

where:

$$A_0 = \frac{1}{T}\int_0^T f(x)dx$$

$$A_m = \frac{2}{T}\int_0^T f(x)\cos\left(\frac{\pi m x}{T}\right)dx$$

SECTION 10.5: THE HEAT EQUATION

The best advice I can give you is: Read the PDE handout, specifically the section about the heat equation! It outlines all the important steps you'll need!

Also read the **important note** I wrote at the beginning of the previous section!

And again, no cheating, do it from scratch, and use tabular integration to evaluate the integrals!

10.5.7. Before you do this problem, use the following trick (not on the exam):
Define $v(x) = 5 + \frac{5}{\pi}x$ and let $w(x,t) = u(x,t) - v(x)$. Then find the PDE that w solves, solve it, and then use $u(x,t) = w(x,t) + 5 + \frac{5}{\pi}x$

SECTION 10.6: THE WAVE EQUATION

Read the PDE handout, specifically the section about the wave equation! It outlines all the important steps you'll need!

And again, no cheating, do it from scratch, and use tabular integration to evaluate the integrals!

10.6.3. Before you use tabular integration, it might be helpful to write $x^2(\pi - x)$ as $\pi x^2 - x^3$